$$y^{\frac{n+1}{n-1}}\left[\frac{1}{2}B\left(\frac{1}{2},\frac{n}{n+1}\right)+2^{\frac{n+1}{n-1}}\left\{B_{\frac{y+\beta}{y}}\left(\frac{n}{n-1},\frac{n}{n-1}\right)-B_{\frac{1}{2}}\left(\frac{n}{n-1},\frac{n}{n-1}\right)\right\}\right]=\left[\frac{2(n+1)}{n-1}\right]^{\frac{1}{n-1}},$$

where $B_p(p, q)$ is the incomplete beta function.

Results calculated with the equations derived are shown in Figs. 2 and 3. Figure 2 shows dimensionless velocity profiles for various values of n and the injection parameter (suction) β [1) n = 2; 2) 1.25; 3) 0.5]. Figure 3 shows the position of the front of the shear perturbations as a function of the injection parameter (suction) β for various values of the rheological constant n [1) n = 2; 2) 1.25; 3) 1.15].

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LITERATURE CITED

1. Z. P. Shul'man and B. M. Berkovskii, The Boundary Layer of Non-Newtonian Liquids [in Russian], Nauka i Tekhnika, Minsk (1966).
2. W. Wilkinson, Non-Newtonian Fluids, Pergamon, New York (1960).
3. O. A. Oleinik, "A system of equations of boundary-layer theory," Zh. Vychisl. Mat. Mat. Fiz., No. 3 (1963).
4. A. I. Suslov, "A system of equations of a boundary layer with a surface of discontinuity," Usp. Mat. Nauk, 29, No. 3 (1974).

VISCOUS EXPLOSION DURING THE NONISOTHERMAL MOTION OF AN INCOMPRESSIBLE LIQUID

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The hydrodynamic thermal explosion of an incompressible liquid moving under pressure in pipes was predicted theoretically in [1-3].

We present the hydraulic theory of a viscous explosion, which is caused by the nonlinear temperature dependence of viscosity.

Let us consider the laminar motion of an incompressible liquid in a circular pipe of radius R and length L. The pressure is p_1 at the pipe inlet and p_2 at the outlet. The temperature of the liquid at the inlet cross section is T_0, and a steady heat flux $\lambda \partial T/\partial r = q_w < 0$ is specified at the pipe walls (heat is removed from the liquid). The physical quantities λ, ρ, and C_p are assumed constant in the temperature range considered.

It is assumed that the Peclet number $Pe = uR\rho C_p/\lambda \gg 1$, so that axial heat conduction can be neglected in the heat-balance equation. We linearize the convective terms of this equation in the following way [4]:

$$\mathbf{v}\,\mathrm{grad}\,T \approx \frac{Q}{\pi R^2}\frac{\partial T}{\partial x},\quad Q = -\pi\int_0^R \frac{du}{dr}r^2 dr.$$

Thus, we consider the equation

$$\frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{\rho C_p Q}{\pi R^2}\frac{\partial T}{\partial x}.$$

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We introduce the temperature of the liquid averaged over the cross section

$$\overline{T}(x) = \frac{2}{R^2}\int_0^R T(x, r)\, r dr.$$

Taking account of the boundary conditions, we obtain for $\overline{T}(x)$

$$\overline{T} = T_0 + \frac{2\pi q_W R}{\rho C_p Q} x.$$

In addition, we assume that the tangential shear stress τ and the velocity gradient du/dr are related by a rheological power law [5]

$$\tau = K\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr},$$

where K is the consistency index and n is the index of non-Newtonian behavior.

Taking account of the experimental data [5], we consider a temperature range in which $K = K(T)$, $n = \text{const}$.

The equation of motion, written for average quantities, has the form [3]

$$\frac{d\bar{p}}{dx} = \frac{1}{r}\frac{d}{dr}\left(r\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr}\right)K(\overline{T}). \tag{1}$$

We note that the assumptions under which Eq. (1) was derived are identical with those of [3], and we shall not discuss them.

From (1)

$$\Delta p = \frac{2}{R^{3n+1}}\left(\frac{3n+1}{n}\frac{Q}{\pi}\right)^n \overline{K}, \tag{2}$$

where $\Delta p = \frac{p_1 - p_2}{L}$; $\overline{K} = \frac{1}{L}\int_0^L K(x)\, dx$.

It follows from (2) that the pressure drop is proportional to Q^n and to the consistency index averaged over the volume of the medium.

Since $K = \overline{K}(Q)$ increases with decreasing flow rate of the liquid, a situation is possible when

$$\lim_{Q\to 0} Q^n K(Q) > 0. \tag{3}$$

This means that the flow curve (2) has a minimum Δp^* for $0 \leq Q < \infty$. In the range $\infty > \Delta p > \Delta p^*$ there are two values of the flow rate of the liquid for each value of the pressure drop; for $\Delta p < \Delta p^*$ there are no steady flows. Physically, this means that heat reaching the pipe walls by convection does not suffice to maintain a steady heat balance.

If a difference arises between the convection of heat in the flowing liquid along the pipe axis and the outflow of heat through the wall, the temperature of the liquid falls, and this in turn leads to an increase in the viscosity and a decrease in the delivery of heat as a result of motion, and to a still larger decrease in temperature, etc. The viscosity of the medium gradually begins to increase with time, and this process can be defined as a viscous explosion. It follows from (3) that the phenomenon described can occur only if the viscosity is strongly temperature dependent.

The phenomenon of viscous explosion clearly cannot occur for such media as water, mercury, etc.

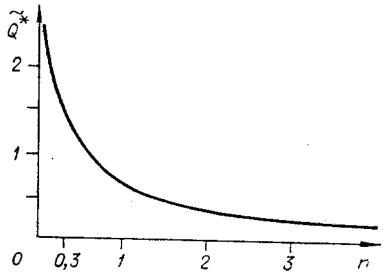

Fig. 1

Let us discuss a specific example. Let $K = K_0 \exp[-\beta(T - T_0)]$, where K_0 and β are constants. Then we obtain from (2)

$$\Delta p = 2\left(\frac{3n+1}{n\pi}\right)^n \frac{Q^{n+1}}{R^{3n+1}} \frac{K_0\rho C_p}{2\pi\beta\,|q_w|\,RL}\left(\exp\frac{2\pi\beta\,|q_w|\,RL}{\rho C_p Q} - 1\right)$$

or in dimensionless form

$$\tilde{p} = \tilde{Q}^{n+1}\left(\exp\frac{1}{\tilde{Q}} - 1\right), \quad \tilde{Q} = \frac{\rho C_p Q}{2\pi\beta\,|q_w|\,RL}, \qquad (4)$$

$$\Delta p = \tilde{p}\,2\left(\frac{3n+1}{n\pi}\right)^n K_0\left(\frac{2\pi\beta\,|q_w|\,RL}{\rho C_p}\right)^n \frac{1}{R^{3n+1}}.$$

The flow curve (4) has a minimum in the range $0 \leqslant \tilde{Q} < \infty$, and is characterized by the critical values of the parameters $\tilde{p}^*$ and $\tilde{Q}^*$. For a Newtonian liquid (n = 1) these values are $\tilde{p}^* = 1.5$, $\tilde{Q}^* = 0.65$, respectively. The relation $Q^* = Q^*(n)$ is shown in Fig. 1.

For Newtonian media (n = 1) Eqs. (4) have the form

$$\tilde{p} = \tilde{Q}^2\left(\exp\frac{1}{\tilde{Q}} - 1\right), \quad \tilde{Q} = \frac{\rho C_p Q}{2\pi\beta\,|q_w|\,RL}, \quad \Delta p = \tilde{p}\,\frac{16K_0\beta\,|q_w|\,L}{R^3\rho C_p}.$$

If, e.g., glycerin is flowing in a pipe with $T_0 = 50$°C, $q_w = -2\cdot10^4$ kcal/m²·h, and L = 2 m, steady states are impossible for $p_1 - p_2 < 3.8$ bar.

At the pipe outlet under critical conditions of viscous explosion, the temperature of the liquid is $T^* = T_0 - 1/\beta Q^* = 20$°C ($\beta \approx 0.05$).

LITERATURE CITED

1. S. A. Kaganov, "Steady laminar flow of an incompressible liquid in a flat channel and a circular cylindrical pipe, taking account of the heat of friction and the temperature dependence of viscosity," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1962).
2. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "Hydrodynamic thermal explosion," Dokl. Akad. Nauk SSSR, 163, No. 1 (1965).
3. A. G. Merzhanov and A. M. Stolin, "Hydrodynamic analogies of the phenomena of ignition and extinction," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1974).
4. R. A. Meric, "An analytical study of natural convection in a vertical open tube," Int. J. Heat Mass Transfer, 20, No. 4 (1977).
5. B. M. Smol'skii, Z. P. Shul'man, and V. M. Gorislavets, Rheodynamics and Heat Transfer of Nonlinearly Viscoplastic Materials [in Russian], Nauka i Tekhnika, Minsk (1970).